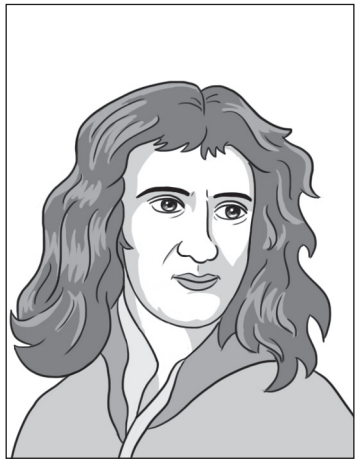
Pan Lloyds Junior Secondary Mathematics New Syllabus Exercises (Book 3)

4. Laws of Indices

Name: _____ Class: _____

Hall of Mathematician

Sir Isaac Newton FRS (25 December 1642 – 20 March 1727), the renowned physicist and mathematician, made significant contributions to the field of mathematics, including his development of the general binomial theorem. By introducing negative exponents into the binomial theorem, Newton extended its applicability to a wider range of mathematical expressions. Newton's work on the general binomial theorem and his use of negative exponents helped establish a foundation for the development of calculus and the study of infinite series.



Sir Isaac Newton FRS

Key Notes

1. Laws of Integral Indices（整數指數定律）

For $a, b \neq 0$ and any integers m and n,

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$

(c) $(a^m)^n = a^{mn}$

(d) $(ab)^m = a^m b^m$

(e) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(f) $a^{-m} = \dfrac{1}{a^m}$ ← We also have $\dfrac{1}{a^{-m}} = a^m$ and $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$.

(g) $a^0 = 1$

2. Scientific Notation（科學記數法）

We can express a large number or a number close to zero in scientific notation $\pm a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

For example:

(a) $2\,4\,5\,0\,0\,0 = 2.45 \times 10^5$ ◄ Move the decimal point 5 places to the left

(b) $-0.0\,0\,6\,1\,7 = -6.17 \times 10^{-3}$ ◄ Move the decimal point 3 places to the right

Remark: If we express an approximated value in scientific notation $a \times 10^n$, then the number of significant figures of that value is same as the number of significant figures of a.

NF 3. Different Numeral Systems（不同進制的記數法）

(a) Denary system（十進制記數法）

In the denary system, the numerals 0, 1, 2, ⋯, 9 are used. The **place values**（位值）of a denary number are 1 (= 10^0), 10^1, 10^2, 10^3, ⋯

For example, $58\,072_{10} = 5 \times 10^4 + 8 \times 10^3 + 0 \times 10^2 + 7 \times 10 + 2 \times 1$

Digit	5	8	0	7	2
Place value	10^4	10^3	10^2	10^1	10^0

(b) Binary system（二進制記數法）

In the binary system, the numerals 0 and 1 are used. The place values of a binary number are 1 (= 2^0), 2^1, 2^2, 2^3, ⋯

For example, $101011_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1 \times 1$

Digit	1	0	1	0	1	1
Place value	2^5	2^4	2^3	2^2	2^1	2^0

(c) Conversion between the denary system and the binary system

(i) From the binary system to the denary system: use an expanded form.

For example, convert 1000101_2 into a denary number.

$1000101_2 = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 64 + 4 + 1$

$= 69_{10}$

(ii) From the denary system to the binary system: use a short division.

For example, convert 25_{10} into binary number.

2	25		
2	12	⋯	1
2	6	⋯	0
2	3	⋯	0
	1	⋯	1

$\therefore 25_{10} = 11001_2$

Concept Check

Determine whether each of the following is correct or not.

1. $2a^0 = 1$ _____

2. $(a^4)^4 = a^{16}$ _____

3. $\left(\frac{2}{3}\right)^5 = \left(\frac{3}{2}\right)^{-5}$ _____

4. $0.003\,42 = 3.42 \times 10^3$ _____

NF **5.** $1010_2 = 1010_{10}$ _____

Step–by–Step Example

Example 1

Express each of the following numbers in scientific notation and round off your answer to 3 significant figures.

(a) 2 398 000

(b) 0.000 521 4

Solution

(a) $2\,398\,000 = 2.398 \times 10^6$

$= \underline{2.40 \times 10^6}$ *(cor. to 3 sig. fig.)*

(b) $0.000\,521\,4 = 5.214 \times 10^{-4}$

$= \underline{5.21 \times 10^{-4}}$ *(cor. to 3 sig. fig.)*

Exercise

Level 0

4.1 Laws of Integral Indices

1. Evaluate each of the following expressions without using a calculator.

(a) 2024^0

(b) 9^{-2}

(c) $6^{-1} \times 6^0$

(d) $(25^0 \times 4)^3$

(e) $(5^{-9} \times 9^5)^0$

(f) $\left(\frac{5^3}{7^0}\right)^{-1}$

2. Simplify each of the following expressions and express your answer in positive indices.

(a) $a^{-2} \times a^6$

(b) $m^{-1}n^7 \times m^3n^{-3}$

(c) $\frac{p^{-9}}{p^{-4}}$

(d) $\frac{x^{-8}y^3}{x^2y^{-7}}$

(e) $(u^5)^{-7}$

(f) $(a^7b^{-4})^{-5}$

(g) $a^{-3}b^0 \times a^0b^{-4}$

(h) $\frac{x^7y^0}{x^0y^{-4}}$

Revision

MC Zone

1. $\dfrac{p^{-8}}{p^{-3}} =$

A. p^{5}.
B. p^{-5}.
C. p^{11}.
D. p^{-11}.

For HKDSE **2.** $9^{n+3} \times 2^{2n+6} =$

A. 18^{n+3}.
B. 18^{3n+9}.
C. 36^{n+3}.
D. 36^{3n+9}.

For HKDSE **3.** $\dfrac{125^{3n+1}}{625^{2n-3}} =$

A. 5.
B. 5^{-n+7}.
C. 5^{n+15}.
D. 5^{17n-9}.

For HKDSE **4.** $\dfrac{3^{3n} \times 9^{4n}}{81^{2n}} =$

A. 3.
B. 3^{n}.
C. 3^{2n}.
D. 3^{3n}.

5. 302 400 000 =

A. 3.024×10^{5}.
B. 3.024×10^{6}.
C. 3.024×10^{7}.
D. 3.024×10^{8}.

6. Which of the following numbers is the greatest?

A. 4.9×10^{5}
B. 4.9×10^{6}
C. 9.4×10^{-5}
D. 9.4×10^{-6}

7. $3.2 \times 10^{-6} - 6.3 \times 10^{-8} =$

A. -3.1×10^{2}.
B. -3.1×10^{-2}.
C. 3.137×10^{-6}.
D. 3.137×10^{-8}.

8. $-0.001\,995 \times 10^{-8} =$

A. -1.995×10^{-11} (correct to 3 significant figures).
B. -2×10^{-11} (correct to 3 significant figures).
C. -2.00×10^{-10} (correct to 3 significant figures).
D. -2.00×10^{-11} (correct to 3 significant figures).

NF **9.** $63_{10} =$

A. 11101_{2}.
B. 111101_{2}.
C. 11111_{2}.
D. 111111_{2}.

NF For HKDSE **10.** $1000100000100100_{2} =$

A. $17 \times 2^{11} + 18$.
B. $17 \times 2^{11} + 36$.
C. $17 \times 2^{12} + 18$.
D. $17 \times 2^{12} + 36$.

NF For HKDSE **11.** $5 \times 2^{8} + 2^{7} - 3 \times 2^{6} =$

A. 10001100000_{2}.
B. 10010000000_{2}.
C. 10011000000_{2}.
D. 10110000000_{2}.

Hint

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CQ Zone

for HKDSE **1.** Simplify each of the following expressions and express your answer in positive indices.

(a) $\dfrac{(m^3n^{-4})^4}{m^{-6}}$

(b) $(a^2b^5) \times (a^{-5}b^6)^4$

2. Express 4 235 600 004 in scientific notation and round off your answer to 3 significant figures.

3. Find the value of $\dfrac{2.4 \times 10^{-4} \times 10\%}{1.8 \times 10^{-7}}$ and express your answers in scientific notation. (Give your answer correct to 3 significant figures.)

NF **4.** Convert 1011101_2 into a denary number.

NF **5.** Convert 107_{10} into a binary number.