# Pan Lloyds Junior Secondary Mathematics New Syllabus Exercises (Book 2)

## 2. Identities and Factorization

Name: _____    Class: _____

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### 🏛 Hall of Mathematician

Leonhard Euler (15 April 1707 – 18 September 1783) was a Swiss mathematician, physicist, astronomer, geographer, logician, and engineer. He founded the studies of graph theory（圖論）and topology（拓撲學）in mathematics. He also made pioneering discoveries in many other branches of mathematics including analytic number theory（解析數論）, complex analysis（複分析）, and infinitesimal calculus（微積分學）.

The following formula is known as Euler's identity.

$$e^{i\pi} + 1 = 0$$

This formula is named after Leonhard Euler. It is a special case of Euler's formula:

$$e^{ix} = \cos x + i \sin x$$

Leonhard Euler

when evaluated for $x = \pi$. Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics.

## Key Notes

### 1. Meaning of Identities

**(a)** Definition

If there is an algebraic equation that holds for any values of the unknowns of the equation, then the equation is called an **identity**（恆等式）. Such relation is expressed with the identity symbol '$\equiv$'.

**(b)** Finding coefficients in an identity

Coefficients in an identity can be found by comparing the coefficients of the like terms and the constant terms on both sides of the identity.

Example: When $Ax^2 - 3x + C \equiv 4x^2 + Bx - 7$, by comparing the coefficients of $x^2$, $x$ and the constant terms, we get $A = 4$, $B = -3$ and $C = -7$.

### 2. Some Important Identities

**(a)** Difference of two squares identity

$$(a + b)(a - b) \equiv a^2 - b^2$$

**(b)** Perfect square identities

$$(a + b)^2 \equiv a^2 + 2ab + b^2$$
$$(a - b)^2 \equiv a^2 - 2ab + b^2$$

### 3. Factorization of Simple Expressions

**(a)** **Factorization**（因式分解）

Factorization is a process of breaking down an expression as a product of 2 or more monomials or polynomials.

**(b)** Methods of Factorization

**(i)** Taking out common factors

Example: $2ab + 4ac = 2a(b + 2c)$

**(ii)** Grouping terms

Example:   $3a + 4b + 6ax + 8bx$

$\qquad = (3a + 4b) + 2x(3a + 4b)$

$\qquad = (3a + 4b)(1 + 2x)$

**(iii)** Using identities

Example: $4a^2 - 9 = (2a + 3)(2a - 3)$

Example: $4x^2 - 4x + 1 = (2x - 1)^2$

### Concept Check

Determine whether each of the following is correct or not.

1. $a^2 + b^2 = (a + b)^2$ is an identity. _____

2. $(2x - 3)^2 = (3 - 2x)^2$ is an identity. _____

3. $9x^2 - 25y^2 = (9x + 25y)(9x - 25y)$ _____

4. $y^2 - 1 = (y - 1)^2$ is an identity. _____

5. $4a^2 + b^2$ cannot be factorized. _____

### Step−by−Step Example

## Example 1

Prove that $2x + 4 = 2(x + 2)$ is an identity.

**Solution**

R.H.S. $= 2(x + 2)$

$\quad\quad = 2x + 4$

$\quad\quad = $ L.H.S.

$\therefore\ 2x + 4 \equiv 2(x + 2)$

**Instant Practice 1**

Prove that $3x - 6 = 3(x - 2)$ is an identity.

**Solution**

R.H.S. $= 3(\quad\quad\quad)$

$\quad\quad = (\quad) - (\quad)$

$\quad\quad = $

$\therefore$

## Example 2

It is given that $6(x + 3) \equiv Ax + B$, where $A$ and $B$ are constants. Find the values of $A$ and $B$.

**Solution**

L.H.S. $= 6(x + 3)$

$\quad\quad = 6x + 18$

$\therefore\ 6x + 18 \equiv Ax + B$

By comparing like terms, we have

$\quad\quad A = \underline{6}$ and $B = \underline{18}$.

**Exercise**

## Level 0

### 5.1 Rate

**1.** Stanley drove 150 kilometers in 2 hours. Find his driving speed in km/h.

**2.** The cost of buying 200 clips is $50. Find the price rate in $/clip.

### 5.2 Ratio

**3.** Simplify the ratio 18 : 24.
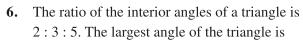
**4.** If $a : b = 2 : 7$ and $b : c = 7 : 5$, find $a : b : c$.

**5.** If $a : b = 2 : 7$ and $b : c = 8 : 3$, find $a : b : c$.

## Revision

### MC Zone

**1.** If $a : b = 7 : 4$ and $b : c = 4 : 7$, then $a : c =$

   A.  1 : 1.

   B.  1 : 1.75.

   C.  1.75 : 1.

   D.  7 : 4.

**2.** If $4x - 9y = 6x - 12y$, then $x : y =$

   A.  8 : 11.

   B.  11 : 8.

   C.  2 : 3.

   D.  3 : 2.

**3.** If a straight line of 8 cm on a map represents a straight road of actual length 2.4 km, then the scale of the map is

   A.  1 : 0.3.

   B.  1 : 30.

   C.  1 : 30 000.

   D.  1 : 300 000.

**4.** It is given that $Y$ is directly proportional to $X$. When $Y = 20$, $X = 8$. When $Y = 80$, then the value of $X$ is

   A.  2.

   B.  32.

   C.  120.

   D.  200.

**5.** It is given that $Y$ is inversely proportional to $X$. When $Y = 48$, $X = 12$. When $X = 72$, the value of $Y$ is

   A.  8.

   B.  12.

   C.  18.

   D.  288.

**6.** The ratio of the interior angles of a triangle is $2 : 3 : 5$. The largest angle of the triangle is

   A.  $40°$.

   B.  $60°$.

   C.  $90°$.

   D.  $100°$.

**7.** If $a : b = 2 : 3$, then $(3a - b) : (3a + 2b) =$

   A.  1 : 2.

   B.  1 : 4.

   C.  2 : 1.

   D.  4 : 1.

**8.** *(for HKDSE)* The costs of powder A and powder B are $12/kg and $20/kg respectively. If $x$ kg of powder A and $y$ kg of powder B are mixed so that the cost of the mixture is $18/kg, find $x : y$.

   A.  1 : 3

   B.  1 : 4

   C.  3 : 1

   D.  4 : 1

Hint

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## CQ Zone

1.   Simplify the ratio 3 km : 500 m.

2.   If 300 km/h = $y$ m/min, find the value of $y$.

3.   If $3a - 2b = 0$ and $b : c = 3 : 5$, find $a : b : c$.

4.   If $x : 3 = (2x + 6) : 15$, find the value of $x$.