6 Locus

Smart Review

A. Different Forms of Equations of Straight Lines

1. **Point-slope form**（點斜式）

    The equation of the straight line passing through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$.

    For example, if a straight line passes through $(2, 1)$ with slope 3, the equation is

    $$y - 1 = 3(x - 2) \quad \blacktriangleleft \text{ Simplify your answer.}$$
    $$3x - y - 5 = 0$$

2. **Two-point form**（兩點式）

    The equation of the straight line passing through two points (x_1, y_1) and (x_2, y_2) is $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

    For example, if a straight line passes through $(4, 5)$ and $(7, 2)$, the equation is

    $$\frac{y - 5}{x - 4} = \frac{2 - 5}{7 - 4}$$
    $$3(y - 5) = -3(x - 4) \quad \blacktriangleleft \text{ Interchanging the values of } x_1 \text{ and } x_2, y_1 \text{ and}$$
    $$x + y - 9 = 0 \qquad\qquad y_2 \text{ simultaneously will get the same result.}$$

    Note: The term $\dfrac{y_2 - y_1}{x_2 - x_1}$ is actually the slope of the line.
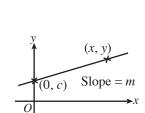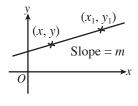
3. **Slope-intercept form**（斜截式）

    The equation of a straight line with y-intercept c and slope m is
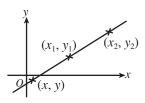
    $$y = mx + c.$$

    For example, the equation of the straight line with y-intercept 4 and slope 5 is $y = 5x + 4$.

    Note: The word "intercept" in slope-intercept form refers to the y-intercept only. When the x-intercept is given instead, we should use point-slope form.

4. **Intercept form**（截距式）

The equation of the straight line with x-intercept a and y-intercept b is

$$\frac{x}{a} + \frac{y}{b} = 1.$$

For example, the equation of the straight line with x-intercept and y-intercept as 1 and 2 respectively is

$$\frac{x}{1} + \frac{y}{2} = 1$$
$$2x + y = 2$$

B. General Form

1. The **general form**（一般式） of the equation of a straight line is $Ax + By + C = 0$, where A, B and C are constants.

    From the general form (when $A, B \neq 0$), we have
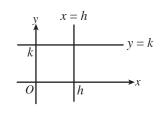
    (a) slope $= -\dfrac{A}{B}$

    (b) x-intercept $= -\dfrac{C}{A}$

    (c) y-intercept $= -\dfrac{C}{B}$

    For example, for the straight line $2x + 7y - 8 = 0$,

    $$\text{slope} = -\frac{2}{7}, \; x\text{-intercept} = 4, \; y\text{-intercept} = \frac{8}{7}.$$

2. When $A = 0$ and $B \neq 0$, the equation becomes $By + C = 0$. It represents a horizontal line with slope zero. It is usually expressed as $y = k$, where k is the y-intercept.

3. When $B = 0$ and $A \neq 0$, the equation becomes $Ax + C = 0$. It represents a vertical line and with slope undefined. It is usually expressed as $x = h$, where h is the x-intercept.

4. When $C = 0$ and $A, B \neq 0$, it is a straight line with slope $-\dfrac{A}{B}$ and it passes through the origin.

5. If (p, q) lies on the straight line $Ax + By + C = 0$, then $Ap + Bq + C = 0$.

    Note: The answers about the equations of straight lines in this chapter are written in general form unless otherwise specified.

Cross Reference

For basic knowledge on coordinate geometry, please refer to Chapter 18.

sample

2. If the equation of a circle is expressed as
$x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants,

it is called the general form. In such case, the centre $= \left(-\dfrac{D}{2}, -\dfrac{E}{2}\right)$

and the radius $= \sqrt{\left(\dfrac{D}{2}\right)^2 + \left(\dfrac{E}{2}\right)^2 - F}$.

For example, for the circle $x^2 + y^2 - 4x - 8y + 3 = 0$, the centre $= (2, 4)$
and the radius $= \sqrt{(-2)^2 + (-4)^2 - 3} = \sqrt{17}$ units.

⊹Point to Remember⊹

To find the centre and the radius of a circle from the general form, we should first make the coefficients of the x^2 term and the y^2 term to be 1.

NF **F. Intersection of a Straight Line and a Circle**

1. Given the equations of a straight line and a circle:

$$\begin{cases} y = mx + c \\ x^2 + y^2 + Dx + Ey + F = 0 \end{cases}$$

By using the method of substitution, a quadratic equation in one unknown $(1 + m^2)x^2 + (2mc + D + Em)x + (c^2 + Ec + F) = 0$ is obtained. The point(s) of intersection can thus be found.

2. Number of points of intersection.

Consider the discriminant Δ of the quadratic equation obtained.

(a) If $\Delta > 0$, then there are 2 points of intersection.

For example,

$x^2 + y^2 - 6x - 6y + 2 = 0$ and $2x - 3y - 4 = 0$

have 2 points of intersection.

HKDSE Tips

Problems about straight line, tangent and circle are frequently asked in HKDSE. Please refer to Q48 and Q50 on P.111 and P.112.

(b) If $\Delta = 0$, then there is 1 point of intersection.

For example,

$x^2 + y^2 - 8x - 6y - 4 = 0$ and $2x + 5y - 52 = 0$

have only 1 point of intersection.

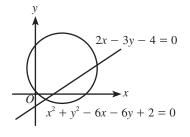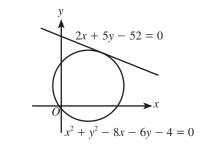(c) If $\Delta < 0$, then there is no point of intersection.

For example,

$x^2 + y^2 - 10x - 8y + 6 = 0$ and $3x + 7y + 32 = 0$

have no point of intersection.

Concept Builder

Determine whether each of the following is true or false.

1. The slope of the line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\dfrac{x_2 - x_1}{y_2 - y_1}$ or $\dfrac{x_1 - x_2}{y_1 - y_2}$.

2. The slope of a horizontal line is 0.

3. If the slopes of two straight lines are equal, then they must have infinite number of points of intersection.

4. The x-intercept of L: $bx + ay + c = 0$ is $-\dfrac{c}{a}$.

5. If $ax + by + c = 0$ and $dx + ey + f = 0$ have no points of intersection, then $ae = bd$.

6. If the centre and the radius of a circle are $(1, 2)$ and 3 units respectively, the equation of the circle is $(x + 1)^2 + (y + 2)^2 = 3^2$.

7. The radius of the circle $4(x + 1)^2 + 4(y + 2)^2 = 6^2$ is 3 units.

8. The centre of the circle $2x^2 + 2y^2 - 6x + 8y + 4 = 0$ is $(3, -4)$.

9. If the distance between the centre of a circle and a point is smaller than its radius, the point should lie outside the circle.

NF 10. The circle $x^2 + y^2 - 25 = 0$ and the straight line $y = 4$ have two points of intersection.

NF 11. The circle $x^2 + y^2 - 4y = 0$ and the straight line $y = x$ have 1 point of intersection.

Worked Examples

Section A(1)

1. The coordinates of A and B are $(8, -7)$ and $(-2, 5)$ respectively. Find the equation of the perpendicular bisector of AB. (4 marks)

Solution

Slope of $AB = \dfrac{5 - (-7)}{-2 - 8} = \dfrac{12}{-10}$

$\qquad\qquad = -\dfrac{6}{5}$

Slope of the perpendicular bisector $= -1 \div \left(-\dfrac{6}{5}\right)$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{5}{6}$ [1M]

> **Point to Remember**
> The perpendicular bisector of AB is the line which is perpendicular to AB and passing through the mid-point of AB.

> Try Q.5.

> **Point to Remember**
> If L_1 is perpendicular to L_2, then $m_{L_1} \times m_{L_2} = -1$.

Concept Builder Ans: 1. F   2. T   3. F   4. F   5. T   6. F   7. T   8. F   9. F
                  10. T   11. F

4. Draw a box-and-whisker diagram for the following set of data.

    1, 3, 3, 4, 5, 6, 7, 7, 7, 8, 10, 12, 12, 13

5. The table shows the airtime (in min) of the calls to a service hotline today.

Time (min)	$1 - 5$	$6 - 10$	$11 - 15$	$16 - 20$
Frequency	34	21	16	9

Find, correct to 3 significant figures, the standard deviation of the airtime. Show your working steps.

NF 6. If the range, the inter-quartile range and the standard deviation of the data set $\{p, q, r, s\}$ are 80, 45 and 22 respectively, find the range, the inter-quartile range and the standard deviation of each of the following data sets.

    (a) $\{p + 3, q + 3, r + 3, s + 3\}$ (b) $\{2p, 2q, 2r, 2s\}$

NF 7. The mean and the standard deviation of the marks of the students in an examination are 60 and 5 respectively. If Samson got 43 in the examination, find the standard score of his mark.

NF 8. In a set of normally distributed data, the mean is 40 and the standard deviation is 5. Find the percentage of data which are between 35 and 45.

⏱ Mock Questions

In the following questions, unless otherwise specified, numerical answers must be either exact or correct to 3 significant figures.

Section A(1)

1. For the set of data 4, 5, 7, 3, 2, 5, 9, find
    (a) the mean;   (b) the range;   (c) the standard deviation.   (3 marks)

2. (a) Find the range, the inter-quartile range and the standard deviation of the following set of data.
        23, 44, 15, 22, 34, 87
   (b) Explain why the inter-quartile range is a more appropriate measure of dispersion than the standard deviation in this case.

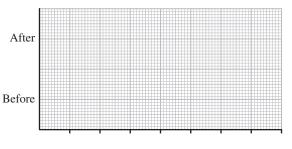   (4 marks)

3. The stem-and-leaf diagram shows the numbers of push-ups finished by a group of students in one minute. Find the median, the inter-quartile range and the standard deviation.

    Reference: HKCEE 07Q4   (3 marks)

Stem (tens)	Leaf (units)
2	3 5 6 7
3	1 3 3 4 5 7 8 8 9
4	2 3 4 6 8
5	0 1

29. The following shows the weights (in kg) of 10 people before and after joining a fitness program run by a fitness centre.

Weight (in kg) before joining the program	Weight (in kg) after joining the program
85　80　90　90　88	73　65　76　102　111
100　110　130　120　100	120　110　65　68　110

(a) Present the above data using two box-and-whisker diagrams in the following graph paper.　　(4 marks)

Weights of people before and after joining a fitness program

Weight (kg)

(b) Find the mean weight before joining the program and the mean weight after joining the program.

(2 marks)

♙**(c)** The manager of the fitness centre claims that by joining the program, all participants can lose at least 10% of weight. Do you agree with this claim? Explain your answer.　　(2 marks)

NF Section B

30. Suppose the mean, the inter-quartile range and the standard deviation of the data set $\{a, b, c, d, e\}$ are 88, 24 and 13 respectively. For the data set $\{3a - 10, 3b - 10, 3c - 10, 3d - 10, 3e - 10\}$, find

(a) the mean;　　(1 mark)

(b) the inter-quartile range;　　(1 mark)

(c) the standard deviation.　　(1 mark)

♙**31.** In a school, the mean of the distribution of the heights of students is 162 cm. The standard scores of Patty and Joe are 1.2 and -2.4 respectively. The height of Joe is 150 cm. A teacher claims that the range of the distribution of the height is at most 15 cm. Is the claim correct? Explain your answer.　　(3 marks)

Reference: HKDSE 16Q16

32. The following shows the records (in m) of Kitty in 10 trials of long jump.

6.2　6.3　6.5　5.8　5.6　7.1　6.3　6.4　6.6　5.9

(a) Find the inter-quartile range and the standard deviation of the records.　　(2 marks)

(b) If the record '7.1 m' is set to be zero due to disqualification,

(i) find the new inter-quartile range;

♙**(ii)** without doing calculation, state the change of the standard deviation.

(3 marks)